Notes Section 6.4 Polar Coordinates

Polar Coordinate System

A **polar coordinate system** is a plane with a point O, the **pole**, and a ray from O, the **polar axis**, as shown in Figure 6.35. Each point P in the plane is assigned **polar coordinates** (r, θ) as follows: r is the **directed distance** from O to P, and θ is the **directed angle** whose initial side is the polar axis and whose terminal side is the ray $\overrightarrow{OP}$.

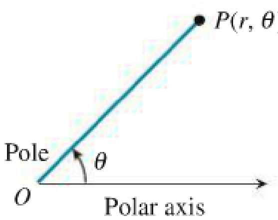


FIGURE 6.35 The polar coordinate system.

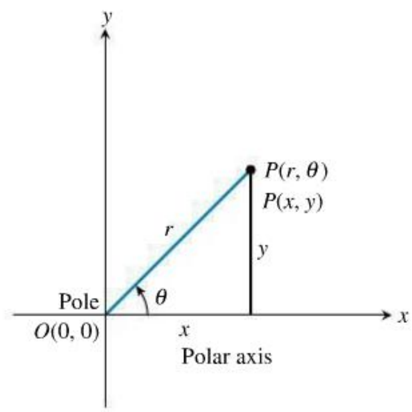


FIGURE 6.38 Polar and rectangular coordinates for P.

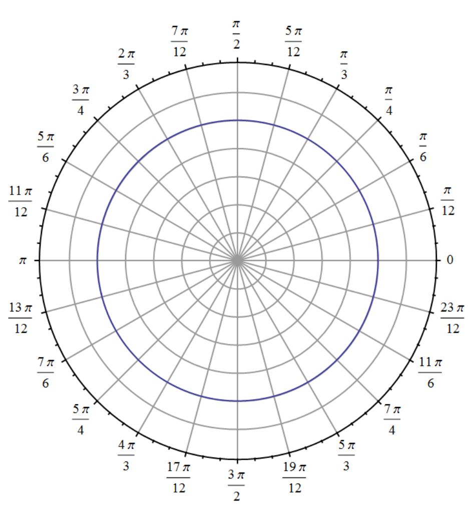
Coordinate Conversion Equations

Let the point P have polar coordinates (r, θ) and rectangular coordinates (x, y). Then

$$x = r\cos\theta, \qquad r^2 = x^2 + y^2,$$

$$y = r\sin\theta, \qquad \tan\theta = \frac{y}{x}.$$

Polar Coordinate Plane



Converting from Polar to Rectangular Coordinates

Ex 1) Use an algebraic method to find the rectangular coordinates of the point with the given polar coordinates. Approximate the exact solution values with a calculator when appropriate (Round two decimal places)

a) $\left(3, \frac{4\pi}{3}\right)$

b) $(-3, -29\pi/7)$

c) $(-2, 120°)$

d) $(-2, \pi)$

Ex 2) Rectangular coordinates of a point P are given. Use an algebraic method, and approximate exact solutions with a calculator when appropriate, to find all polar coordinates of P that satisfy the given condition.

a) $[0, 2\pi]$

P = (1, 1)

b) $-\pi \leq \theta \leq \pi$

P = (1, 3)

Ex 3) Convert the polar equation to rectangular form.

a) $r = 3\sec\theta$

b) $r = -3\sin\theta$

Ex 4) Convert the rectangular equation to polar form.

a) $x = 2$

b) $2x - 3y = 5$

c) $(x-3)^2 + y^2 = 9$

d) $(x+3)^2 + (y+3)^2 = 18$